

Superfluid transition in quasi2D Fermi gases

D.S. Petrov^{1,2}, M.A. Baranov^{2,3}, and G.V. Shlyapnikov^{1,2,4}

¹ *FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands*

² *Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*

³ *Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*

⁴ *Laboratoire Kastler Brossel, 24 rue Lhomond, F-75231 Paris Cedex 05, France*

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We show that atomic Fermi gases in quasi2D geometries are promising for achieving superfluidity. In the regime of BCS pairing for weak attraction, we calculate the critical temperature T_c and analyze possibilities of increasing the ratio of T_c to the Fermi energy. In the opposite limit, where a strong coupling leads to the formation of weakly bound quasi2D dimers, we find that their Bose-Einstein condensate will be stable on a long time scale.

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Recent progress in trapping and cooling of Fermi isotopes of K [1,2] and Li [3,4,5,6] has shown the ability to go far below the temperature of quantum degeneracy and to manipulate independently the trapping geometry, density, temperature and interparticle interaction. The Duke experiment [7] presents intriguing results on the possibility of achieving a superfluid phase transition in the two-component Fermi gas of ^6Li .

Two-dimensional Fermi gases have striking features not encountered in 3D. In the superfluid state, thermal fluctuations of the phase of the order parameter strongly modify the phase coherence properties. The interaction strength depends logarithmically on the relative energy of the colliding atoms. For degenerate Fermi gases this energy is of the order of the Fermi energy ε_F which is proportional to the 2D density n . Accordingly, the exponential dependence of the BCS transition temperature on the interaction strength transforms into a power law dependence on the density: $T_c \propto n^{1/2}$ [13,14]. This suggests a unique possibility to cross the critical point by adiabatically expanding a degenerate Fermi gas. Since the ratio T/ε_F remains unchanged, the temperature scales as n and decreases with density faster than T_c .

Experimentally it is possible to achieve the quasi2D regime by confining the atoms in one direction so tightly that the corresponding level spacing exceeds the Fermi energy. Under this condition the degenerate Fermi gas is kinematically two-dimensional. Thus far, this regime has been reached for Cs atoms [8,9,10] and for Bose-Einstein condensates of Na [11] and Rb [12].

In the quasi2D regime the mean-field interaction between particles exhibits a similar logarithmic dependence on the particle energy as in the purely 2D case [15]. The amplitude of the s -wave scattering turns out to be sensitive to the strength of the tight confinement [15]. This opens new handles on manipulations of the interparticle interaction and superfluid pairing.

In this Rapid Communication we show that atomic Fermi gases in quasi2D geometries can become strong

competitors of 3D gases in achieving superfluidity. The ability to increase the interparticle interaction by tuning the trap frequencies gives an opportunity to realize a transition from the standard BCS pairing in the case of weak attraction to the limit of strong interactions and pairing in coordinate space. In the latter case one eventually gets a dilute system of weakly bound quasi2D dimers of fermionic atoms, which can undergo Bose-Einstein condensation. For the BCS case, we calculate the critical temperature T_c to second order in perturbation theory and discuss possibilities of increasing the ratio T_c/ε_F . In the other extreme, we find that the interaction between the quasi2D dimers is repulsive, and their collisional relaxation and decay are strongly suppressed. This allows us to conclude that BEC of these composite bosons will be stable on a long time scale.

We consider an ultracold two-component Fermi gas in the quasi2D regime and confine ourselves to the s -wave interaction and superfluid pairing between atoms of different components. We assume that the characteristic radius of the interaction potential is much smaller than the harmonic oscillator length in the tightly confined direction, $l_0 = (\hbar/m\omega_0)^{1/2}$, where m is the atom mass, and ω_0 is the confinement frequency. Then the interaction problem involves two length scales: l_0 and the 3D scattering length a . For $a < 0$ and $|a| \ll l_0$, there is a peculiar quasi2D weakly bound s -state of two particles, with the binding energy [15]

$$\varepsilon_0 = 0.915(\hbar\omega_0/\pi) \exp(-\sqrt{2\pi}l_0/|a|) \ll \hbar\omega_0. \quad (1)$$

In this case the coupling constant for the intercomponent interaction takes the form $g = (4\pi\hbar^2/m) \ln^{-1}(\varepsilon_0/\varepsilon)$, where the relative collision energy ε is assumed to be either much smaller or much larger than ε_0 (see [15] and refs. therein). As in degenerate Fermi gases one has $\varepsilon \sim \varepsilon_F$, the interaction is attractive ($g < 0$) if the density is sufficiently high and one satisfies the inequality

$$\varepsilon_0/\varepsilon_F \ll 1. \quad (2)$$

Thus, the inequality (2) is the necessary condition for the BCS pairing. For finding the critical temperature T_c below which the formation of Cooper pairs becomes favorable, we go beyond the simple BCS approach and proceed along the lines of the theory developed by Gor'kov and Melik-Barkhudarov for the 3D case [16].

The critical temperature T_c is determined as the highest temperature for which the linearized equation for the order parameter (gap) $\Delta = \langle g\hat{\Psi}\hat{\Psi} \rangle$ has a nontrivial solution [17]. Assuming that the quasi2D gas is uniform in two in-plane directions, the gap equation in the momentum space takes the 2D form

$$\Delta(\mathbf{q}) \approx - \int \left\{ \frac{\hbar^2}{m} f(z, \mathbf{q}, \mathbf{q}') \left[K(q') + \frac{1}{z - \xi(q') + i0} \right] + \delta V(\mathbf{q}, \mathbf{q}') K(q') \right\} \Delta(\mathbf{q}') \frac{d^2 q'}{(2\pi)^2}, \quad (3)$$

where $K(q) = (1/2\xi(q)) \tanh(\xi(q)/2T)$, $\xi(q) = \hbar^2 q^2/2m - \mu$, and $\mu \approx \varepsilon_F = \pi \hbar^2 n/m$ is the chemical potential. The first term in the rhs of Eq.(3) results from the direct interaction between particles, and we renormalized the interaction potential in terms of the scattering amplitude f (vertex function). The latter is a solution of the quasi2D scattering problem. The parameter z has a meaning of the total energy of colliding particles in their center of mass reference frame. It is of the order of ε_F and drops out of the final answer. The term $\delta V(\mathbf{q}, \mathbf{q}')$ describes the modification of the interparticle interaction due to the presence of other particles (many-body effects). The leading contributions to this term are second order in the scattering amplitude. They are shown in Fig.1 and correspond to an indirect interaction between two particles when one of them interacts with a particle-hole pair virtually created from the ground state (filled Fermi sea) by the other particle. These second order contributions are important for the absolute value of the critical temperature (preexponential factor in the 3D case), whereas higher order terms involving more interaction events can be neglected.

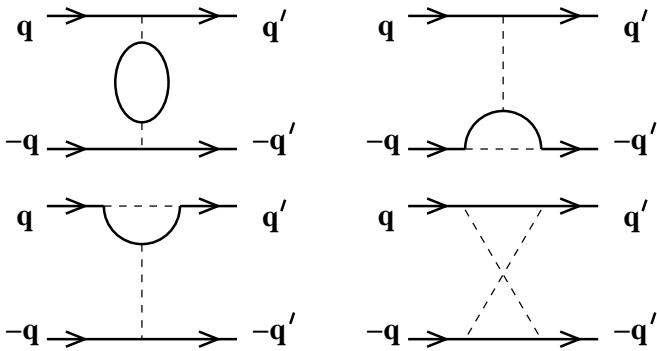


FIG. 1. The leading contributions to $\delta V(\mathbf{q}, \mathbf{q}')$.

The amplitude f is independent of the momenta \mathbf{q}, \mathbf{q}' and we will use the 2D relation (see [18])

$$f = 4\pi \ln^{-1} \{ \varepsilon_0 / (-z) \}. \quad (4)$$

The last term in the rhs of Eq.(3) is a small correction, since $\delta V \sim f^2$. Accordingly, the quantity $|f|$ represents a small parameter of the theory. In the quasi2D regime ($z \ll \hbar\omega_0$) the motion of particles in the tightly confined direction provides a correction to Eq.(4), which is $\sim (z/\hbar\omega_0) f^2$ [15]. It is much smaller than δV and will be omitted.

Equations (3) and (4) show that the momentum dependence of the order parameter appears only due to the second order term that contains many-body contributions to the interparticle interaction δV . The latter is a function of $p = |\mathbf{q} + \mathbf{q}'|$ and rapidly decays for $p > 2q_F$, where $q_F = \sqrt{2m\mu}/\hbar$ is the Fermi momentum. For $p \leq 2q_F$ the quantity δV is almost constant. Therefore, one has $\Delta(\mathbf{q}') \approx \Delta(q_F)$ in a wide momentum range near the Fermi surface. Then, for $q = q_F$ a direct integration of Eq.(3) yields

$$\Delta(q_F) = - \frac{f(z)}{4\pi} \Delta(q_F) \ln \left[\frac{(-2\mu z)}{\pi^2 T^2} \exp(2\gamma) \right] - \delta V(q_F, q_F) \frac{m}{2\pi \hbar^2} \ln \left(C \frac{\mu}{T} \right) \Delta(q_F),$$

where $\gamma \approx 0.5772$ is the Euler constant, and C is a numerical factor determined by the momentum dependence of Δ and δV . The calculation of $\delta V(q_F, q_F)$ is straightforward and gives $\delta V(q_F, q_F) = (\hbar^2/2\pi m) f^2 (2\mu)$. Then, using Eq.(4) we obtain the critical temperature

$$T_c \approx (2\mu/\pi) \exp(\gamma - 1 - |2\pi \text{Re} f^{-1}(2\mu)|).$$

The exponent in this equation should be large and the quantity $\text{Re} f^{-1}(2\mu)$ should be negative. As the chemical potential is $\mu \approx \varepsilon_F$, from Eq.(4) one sees that these requirements are reached under the condition (2). Using Eqs. (1) and (4) the critical temperature takes the form

$$T_c = \frac{\gamma \sqrt{2\varepsilon_0 \varepsilon_F}}{\pi e} = 0.16 \sqrt{\varepsilon_F \hbar \omega_0} \exp \left(- \sqrt{\frac{\pi}{2}} \frac{l_0}{|a|} \right) \ll \varepsilon_F. \quad (5)$$

The relative correction to this result is of the order of $1/|\ln(\varepsilon_0/\varepsilon_F)| \ll 1$.

Note that Eq. (5) predicts by a factor of e smaller value for the critical temperature than a simple BCS calculation [19]. This means that the attractive interaction between particles becomes weaker once we take into account the polarization of the medium.

The ratio T_c/ε_F is not necessarily very small. For example, using Feshbach resonances the scattering length is tunable over a wide interval of negative values [1,5,6]. Keeping the exponential term equal to 0.05 in Eq.(5), with $\omega_0 \sim 100$ kHz we obtain $T_c/\varepsilon_F \sim 0.1$ for 2D densities $n \sim 10^9$ cm $^{-2}$ ($T_c \sim 40$ nK).

As in the purely 2D case [13,14], the transition temperature $T_c \propto n^{1/2}$ and the ratio T_c/ε_F increases with decreasing density as $n^{-1/2}$. This is a striking difference from the 3D case, where this ratio decreases exponentially with density. In the presence of the in-plane confinement, one can approach the BCS transition in a degenerate Fermi gas by adiabatically expanding the quasi2D trap in the in-plane direction(s). As the degeneracy parameter T/ε_F is conserved in the course of the adiabatic expansion, the ratio T/T_c will decrease as $n^{1/2}$. Equations (1) and (5) also show that one can increase ε_0 and T_c/ε_0 by tuning $|a|$ to larger values or by making the tight confinement stronger and thus decreasing l_0 .

What happens if ε_0 and ε_F become comparable with each other, i.e. one reaches the quasi2D resonance for two-body collisions? Then Eq.(5) leads to $T_c \sim \varepsilon_F$ and is no longer valid. In fact, for $\varepsilon_0 > \varepsilon_F$ the formation of bound quasi2D dimers of distinguishable fermions becomes energetically favorable and one encounters the problem of Bose-Einstein condensation of these bosonic molecules. Thus, an increase of the ratio $\varepsilon_0/\varepsilon_F$ from small to large values is expected to provide a transformation of the BCS pairing to molecular BEC. This type of crossover has been discussed in literature in the context of superconductivity [20,21,22,23,14] and in relation to superfluidity in 2D films of ^3He [13,24]. The idea of using a Feshbach resonance for achieving a superfluid transition in the BCS-BEC crossover regime in ultracold 3D Fermi gases has been proposed in refs. [25,26].

We will not consider the crossover regime and confine ourselves to the limiting case of molecular BEC ($\varepsilon_0 \gg \varepsilon_F$). A subtle question is related to the stability of the expected condensate, which depends on the interaction between the molecules. For the repulsive interaction one will have a stable molecular BEC, and the attractive interaction should cause a collapse.

The molecule-molecule scattering is a 4-body problem described by the Schrödinger equation

$$\left[-\frac{\hbar^2}{m} (\nabla_{\mathbf{r}_1}^2 + \nabla_{\mathbf{r}_2}^2) - \frac{\hbar^2}{2m} \nabla_{\mathbf{R}}^2 + U(r_1) + U(r_2) + \sum_{\pm} U\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \pm \mathbf{R}\right) - E \right] \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R}) = 0. \quad (6)$$

Here \mathbf{r}_1 is the distance between two given distinguishable fermions, \mathbf{r}_2 is the distance between the other two, \mathbf{R} is the distance between the centers of mass of these pairs, and U is the interatomic potential. The total energy is $E = -2\varepsilon_0 + \varepsilon$, with ε being the relative molecule-molecule kinetic energy.

The interaction between molecules is present only at intermolecular distances of the order of or smaller than the size of a molecule $d_* = \hbar/\sqrt{m\varepsilon_0}$. Therefore, at energies $\varepsilon \ll \varepsilon_0$ the scattering between molecules is dominated by the s -wave channel and can be ana-

lyzed on the basis of the solution of Eq.(6) for $\varepsilon = 0$. For large R the corresponding wavefunction is $\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R}) \approx K_0(r_1/d_*)K_0(r_2/d_*)\ln(\alpha R/d_*)$, where the decaying Bessel function $K_0(r_{1,2}/d_*)$ represents the 2-body bound state. The parameter α is a universal constant which can be found by matching the quantity $\ln(\alpha R/d_*)$ with the solution of Eq.(6) at short distances. Finally, matching $\ln(\alpha R/d_*)$ with the wavefunction of free relative motion of two molecules at distances $d_* \ll R \ll \Lambda_\varepsilon$, where $\Lambda_\varepsilon = \hbar/\sqrt{m\varepsilon}$ is their de Broglie wavelength, we obtain the coupling constant (scattering amplitude) for the interaction between molecules:

$$g_m = (2\pi\hbar^2/m) \ln^{-1}(2\alpha^2 e^{-2\gamma} \varepsilon_0/\varepsilon) > 0; \quad \varepsilon \ll \varepsilon_0. \quad (7)$$

A precise value of α is not important as it gives rise to higher order corrections in Eq.(7). However, in order to make sure that this constant is neither anomalously large nor anomalously small we have integrated Eq.(6) numerically. For this purpose, it is convenient to transform Eq.(6) into an integral equation for a function which depends only on three independent coordinates. This has been done by using the method of ref. [27]. Our calculations lead to $\alpha \approx 1.6$. They show the absence of 4-body weakly bound states and confirm an intuitive picture that the interaction between two molecules can be qualitatively represented by means of a purely repulsive potential with the range $\sim d_*$. For the interaction between Bose-condensed dimers, in Eq.(7) one has $\varepsilon = 2n_m g_m \ll \varepsilon_0$, where n_m is the density of the dimers (see [15] and refs. therein). We thus conclude that a Bose condensate of these weakly bound dimers is stable with respect to collapse.

The 2D gas of bosons becomes Bose-condensed below the Kosterlitz-Thouless transition temperature T_{KT} [28] which depends on the interaction between particles. According to the recent quantum Monte Carlo simulations [29], for the 2D gas with the coupling constant (7) the Kosterlitz-Thouless temperature is given by

$$T_{KT} = (\pi\hbar^2 n_m/m) \ln^{-1}[(\eta/4\pi) \ln(1/n_m d_*^2)], \quad (8)$$

where the numerical factor $\eta \approx 380$. For $\varepsilon_F \ll \varepsilon_0$, the density of dimers $n_m \approx n/2$ and the parameter $(1/n_m d_*^2) \approx 2\pi\varepsilon_0/\varepsilon_F$. Then, for $\varepsilon_0/\varepsilon_F = 10$, Eq.(8) gives $T_{KT}/\varepsilon_F \approx 0.1$ and at densities 10^8 cm^{-2} the transition temperature in the case of ^6Li is $T_{KT} \approx 30 \text{ nK}$.

The weakly bound dimers that we are considering are molecules in the highest rovibrational state and they can undergo collisional relaxation and decay. The relaxation process occurs in pair dimer-dimer or dimer-atom collisions. It produces diatomic molecules in deep bound states and is accompanied by a release of the kinetic energy. The size of these deeply bound molecules is of the order of the characteristic radius of the interatomic potential $R_e \ll l_0$, and their internal properties are not

influenced by the tight confinement. Therefore, the relaxation can be treated as a 3D process and it requires the presence of at least three fermionic atoms at distances $\sim R_e$ between them. Since at least two of them are identical, the relaxation probability acquires a small factor $(kR_e)^2$ compared to the case of bosons, where $k \sim 1/d_* = \sqrt{\varepsilon_0/\hbar\omega_0}/l_0$ is a characteristic momentum of atoms. The 3D density of atoms in the quasi2D geometry is $\sim n/l_0$. Thus, qualitatively, the inverse relaxation time can be written as $\tau_{\text{rel}}^{-1} \sim \alpha_{\text{rel}} n (R_e/l_0)^2 (\varepsilon_0/\hbar\omega_0)/l_0$, where α_{rel} is the relaxation rate constant for the highest rovibrational states of 3D molecules of two bosonic atoms. We estimate τ_{rel} keeping in mind the recent measurements for Rb_2 molecules [30] which give $\alpha_{\text{rel}} \approx 3 \times 10^{-11} \text{ cm}^3/\text{s}$. For l_0 in the interval from 10^{-5} to 10^{-4} cm , the suppression factor $(R_e/l_0)^2 (\varepsilon_0/\hbar\omega_0)$ ranges from 10^{-3} to 10^{-5} and at 2D densities $n \sim 10^8 \text{ cm}^{-2}$ we find the relaxation time τ_{rel} of the order of a second or larger.

Dimer-dimer pair collisions can lead to the formation of bound trimers, accompanied by a release of one of the atoms. The formation of deeply bound trimer states will be suppressed at least in the same way as the relaxation process discussed above. Therefore, it is important that there are no weakly bound trimers in (quasi)2D. Just as in 3D [31], this can be established by using the zero-range approach ($R_e \rightarrow 0$). We have performed this analysis along the lines of the 3D work [27]. Qualitatively, the symmetry of the 3-fermion system containing two identical fermions provides a strong centrifugal repulsion that does not allow the presence of 3-body bound states. This is in contrast to 2D bosons where one has two fully symmetric trimer bound states [32].

Thus, the life-time of quasi2D dimers of fermionic atoms is rather long and one easily estimates that it greatly exceeds the characteristic time of elastic collisions. One can even think of achieving BEC in the initially non-condensed gas of dimers produced out of a non-superfluid atomic Fermi gas under a decrease of n or l_0 .

In conclusion, we have found the temperature of superfluid phase transition in two-component quasi2D Fermi gases. Our results are promising for achieving this transition in both the regime of BCS pairing and the regime of BEC of weakly bound dimers.

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